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USING PDE MODEL AND SYSTEM DYNAMICS MODEL FOR DESCRIBING MULTI-OPERATION PRODUCTION LINES

O. Pihnastyi
Doctor of Technical Sciences, Professor*
E-mail:pihnastyi@gmail.com
D. Yemelianova
PhD*
E-mail:darya.yemelianova@gmail.com
D. Lysytsia
PhD**
E-mail: lysytsia-mail@ukr.net
*Department of Distributed Information Systems and Cloud Technologies
Department of Computer Engineering and Programming*
***National Technical University «Kharkiv Polytechnic Institute»
Kyrpychova str., 2, Kharkiv, Ukraine, 61002

Two classes of models for describing production flow lines are analyzed. The use of models of these classes for the design of highly efficient control systems of production lines, the technological route of which consists of a large number of technological operations, is analyzed. The division of the technological route into a large number of operations is caused by the development trend of modern production lines. Synchronization of production line equipment performance is provided by an accumulating buffer. A formalized description of the production line was used as a foundation for constructing equations for each models class. The common features of using each models class in the description of production systems, as well as the conditions for their application are shown. The form of the system dynamics model and PDE model equations is substantiated. The assumption about a deterministic rate of processing parts and the absence of a time delay and feedback between the parameters of technological operations was made when deriving the equations. The use of generalized technological operations in the system dynamics model as a way to reduce the number of model equations is discussed. Two limiting transitions from the PDE model equations to the system dynamics equations are demonstrated. It is shown that the system dynamics equations are a special case of the PDE model equations, the result of aggregation of production line parameters within the technological operation. The method for constructing level equations for the system dynamics model is substantiated. For production lines with a different number of operations, the solution to the problem of processing parts along a production line is presented. The comparative analysis of the solutions obtained using the system dynamics and PDE model equations is obtained

Keywords: production line, technological route, system dynamics, PDE production model

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1. Introduction

A production line is a progressive method of organizing production. The processing of the part along the production line occurs in accordance with the given technological route of product manufacturing [1]. The technological route of modern production lines consists of a large number (10^2–10^3) of technological operations. The global trend of a constant increase in the operations number and production lines throughput determines the high relevance of the further development of models of production lines used to design effective production control systems. In [2], a production line of 100 machines, in [3] of 26 machines, in [4] of 300 machines, in [5] of 250 operations and 100 machines, in [6] of 18 machines, in [7] of 10 and 50 machines is considered. The analysis of the papers demonstrates that a multi-operation production line is a complex dynamic distributed system. The increased requirement to improving the accuracy of modeling production systems was directly related to the need for a detailed description of the state of distributed system parameters, which limited the use of many common models for describing production lines. Among the limited range of models that are currently successfully used to describe production lines with parameters distributed along the technological route, two most common classes of models should be distinguished: system dynamics model [8] and PDE model of the production line [9]. Therefore, the improvement of the method for constructing level equations and rate equations of the system dynamics using the PDE model equations is relevant. The improvement makes it possible to use, instead of the phenomenological approach, characteristic for the system dynamics, the statistical approach, which is the foundation of the PDE model, which allows taking into account accurate model concepts of the interaction of parts with equipment and each other as a result of processing.

2. Literature review and problem statement

In [8], the principles of constructing a model of an industrial system using system dynamics equations are given, the structure of the model and six interconnected networks of production activities are determined. The material flow network takes into account the part processing along the technological route. The system of level and rate equations is a tool for describing distributed production systems, technological routes of which consist of a large number of technological operations. The presence of the delay parameter is associated with the methods of dividing the technological route into generalized technological operations [10]. The methods of random cycle diagrams [11] and resource identification [12] do not take into account the structure of stocks and flows of systems. The use of general structures of the subject area [13] and component strategy [14] did not solve this problem either. One of the laborious ways to solve the problem is based on the concept of an interaction matrix that links resource flows [15]. The use of the PDE model equations can be an option to overcome the corresponding difficulties for production lines with a large number of technological operations [16]. The use of partial differential equations has significantly expanded the design capabilities of control systems for production lines [17]. PDE models, as well as system dynamics models, are used to describe a multi-operation production line that operates under steady-state and transient mode [18–20].

3. The aim and objectives of the study

The aim of the study is to compare the use of the system dynamics model and PDE model equations to describe production lines and identify the relationship between the models.

To achieve the aim, the following objectives were set:

– to show that the system dynamics equations are the limiting case of the PDE model equations for a technological route with a large number of operations;

– to demonstrate the results of solving the problem of parts processing along the technological route using the system dynamics model and PDE model equations.

4. Materials and methods of production lines research

4. 1. Formalized description of a production line

A production line at an enterprise with the flow production method is a set of in-line or parallel equipment a_m in accordance with the technological route of product manufacturing. Each m technological operation is a complete part of the technological process [1], performed on the m technological equipment. For the synchronized operation of the production line, in order to prevent downtime of equipment, the m technological operation has a buffer b_m for storing parts awaiting processing. The buffer size is determined by the permissible deviation between the actual and standard performance during the production process [21]. This structure is typical for many industries, including the automotive industry. The analysis of the structure of the production serial line, lines with converging, branching, re-entrant technological routes is presented in [6]. To describe production lines, distributed parameters are used that characterize the capacity of the buffer b_m and performance of processing equipment a_m. The use of these parameters is a common approach when constructing models of production lines [2, 6, 7, 9]. These parameters are used in the system dynamics and PDE model equations.

The structure of the production serial line is used for comparing the two classes of models [6] (Fig. 1). Parts arrive at the first technological operation with equipment a_1, buffer b_1, are processed in successive technological operations. The change in the number of parts in the buffer b_m can be characterized by the parameter $V_m(t)$, the time change in the performance of equipment a_m by the parameter $[\chi]_{1m}(t)$.

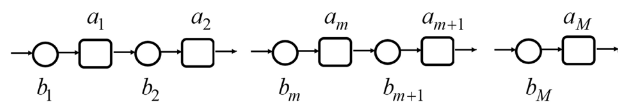

Fig. 1. Serial production line

This structure of the production line is widely used in industry in the form of separate multi-operational sections of the technological route, it allows simplifying the process of formalized description of the production line and interpretation of the results of qualitative analysis of models. Complex branched structures of production lines are represented as a combination of separate serial lines for main and slave products [10].

4. 2. Features of applying system dynamics model equations

The system of the production line equations (Fig. 1), which determines the number of parts $V_m(t)$ in the buffer b_m with a capacity of $V_{\max m}$ for the m technological operation with the processing rate of parts $[\chi]_{1m}(t)$ on the equipment a_m at time t is as follows:

$$\frac{dV_m(t)}{dt}=[\chi]_{1m-1}(t)-[\chi_m]_{1m}(t),\ m=1...M, \tag{1}$$

$$[\chi]_{1m}(t)=[\chi]_{1\psi m}(t), \tag{2}$$

with the number of parts in the buffer b_m at time t=0 equal to $V_m(0)=V_{m0}$. The level equation (1) is supplemented by the rate equation (2) [8]. The performance $[\chi]_{1m}(t)$ of the equipment a_m depends on the conditions and mode of processing, it is characterized by the average rate $[\chi]_{\psi m}(t)$ and standard deviation. When describing the production line, we assume that equality (2) is fulfilled, there are no time delays and feedbacks between the model parameters. The arrival rate of parts at the input of the production line is denoted by $[\chi]_{10}(t)$, the output flow of parts from the production line is $[\chi]_{1M}(t)$. The solution of the system of equations (1), (2) with the given initial conditions determines the distribution of parts by technological operations at time t is as follows:

$$V_m(t)=V_m(t)+\int_0^t\left([\chi]_{1m-1}(\tau)-[\chi]_{1m}(\tau)\right)d\tau,$$

$$m=1...M. \tag{3}$$

For a constant value of the equipment operation rate $[\chi]_{1m}(t)=[\chi]_{1m}$=const at the m operation, solution (4) is as follows

$$V_m(t)=V_m(0)+\left([\chi]_{1m-1}-[\chi]_{1m}\right)t,$$

$$m=1...M. \tag{4}$$

The solution of equations (3), (4) determines the state of the parameters of the production line in the absence of time delays and feedbacks. To reduce the number of equations for M>>1(10^2–10^3), several technological operations are combined into one generalized operation [10].

4. 3. Features of applying PDE model equations

When building a PDE model, the flow of parts along the production route is considered as a continuous flow. We assume that the position of the part in the technological route at the time t satisfies the trajectory of movement $S=S(t)$, which is set in accordance with the route maps for the product manufacture. For the conveyor line of mining and processing companies, the coordinate S determines the path traveled by the material [22], for models of semiconductor lines, the coordinate S corresponds to the degree of completeness of the product manufacturing $S\in[0, 1]$ [23]. The choice of the S coordinate in the form of the cost of resources spent on the manufacture of the product [16] makes it possible to build efficient dynamic distributed models of inventory management. Let us consider the construction of PDE model equations for a production line with the structure shown in Fig. 1. Let's introduce the concept of the density of parts by the m operation along the technological route

$$[\chi_0](t,S_m)=\frac{V_m(t)}{\Delta S_m},$$

$$V_m(t)=\Omega(t,S_m)-\Omega(t,S_{m-1}). \quad (5)$$

The value $\Omega(t, S_m)$ specifies the number of parts that are in the state of processing, $S\in[0, S_m]$ (not processed in the m technological operation). The value $V_m(t)$ is the number of parts in the interval ΔS_m=$\Delta S_m \Delta S_{m-1}$ between the (m1) and m operation in the technological space (S, $\mathbf{\mu}$). Equation (5) defines the relationship between parameters in the system dynamics model and the PDE model. The production line model (Fig. 1) is considered in a one-moment approximation [9, 16, 23]:

$$\frac{\partial[\chi]_0(t,S)}{\partial t}+\frac{\partial[\chi]_1(t,S)}{\partial S}=0, \quad (6)$$

$$[\chi]_1(t,\mathrm{S})=[\chi]_{1\psi}(t,S), \quad (7)$$

with the distribution of parts at the initial moment of time

$$[\chi]_0(0,S)=[\chi]_{0S}(S). \quad (8)$$

As for the system dynamics model, it is assumed that the rate $[\chi]_1(t, S)$ is equal to the average value $[\chi]_\psi(t, S)$, there are no feedbacks between the parameters of the production line. The value of transport delay is determined by equation (6), which is the law of conservation of the number of parts as a result of their processing along the technological route.

Equation (6) can be represented as

$$[\chi]_0(t,S)=[\chi]_0(0,S)-\int_0^t\frac{\partial[\chi]_1(\tau,S)}{\partial S}\mathrm{d}\tau=$$
$$=[\chi]_0(0,S)-\int_0^t\frac{\partial[\chi]_{1\psi}(\tau,S)}{\partial S}\mathrm{d}\tau. \quad (9)$$

For a constant value of the equipment operation rate $[\chi]_{1\psi}(t, S)$=$[\chi]_{1\psi S}(S)$ in the m operation, solution (5) is as follows

$$[\chi]_0(t,S)=[\chi]_0(0,S)-\frac{\partial[\chi]_{1\psi}(S)}{\partial S}t. \quad (10)$$

The value of interoperational backlogs at the m operation $S\in[S_{m-1}, S_m]$ is determined by the expression

$$V_m(t)=\int_{\mathrm{S}_{m-1}}^{\mathrm{S}_m}[\chi]_0(t,\mathrm{S})\mathrm{d}S=$$
$$=[\chi]_0(t,\mathrm{S}_m)\Delta\mathrm{S}_m+0(\Delta\mathrm{S}_m). \quad (11)$$

The solution of equations (6), (7) determines the state of the production flow line parameters.

5. Relationship between system dynamics and PDE model equations

The equations of the system dynamics model (1), (2) and PDE model (6), (7) are examples of discrete and continuous description of a production line [24, 25]. With a discrete description, to reduce the number of level and rate equations, technological operations are combined [11] into a generalized operation. Let us consider the construction of system dynamics equations for a production line with the structure shown in Fig. 1. Let us integrate equation (6) within the m operation

$$\int_{S_{m-1}}^{S_m}\frac{\partial[\chi]_0(t,S)}{\partial t}\mathrm{d}S+\int_{S_{m-1}}^{S_m}\frac{\partial[\chi]_1(t,S)}{\partial S}\mathrm{d}S=0,$$

$$\Delta S_m=\Delta S_m-\Delta S_{m-1}. \quad (12)$$

Taking into account the constraint equation (11), the integration of the first and second terms leads to the following form

$$\int_{S_{m-1}}^{S_m}\frac{\partial[\chi]_0(t,S)}{\partial t}\mathrm{d}S\approx$$
$$\approx\frac{\mathrm{d}}{\mathrm{d}t}\int_{S_{m-1}}^{S_{m-1}+\Delta S_m}[\chi]_0(t,S)\mathrm{d}S=$$
$$=\frac{\mathrm{d}V_m(t)}{\mathrm{d}t}, \quad (13)$$

$$\int_{S_{m-1}}^{S_m}\frac{\partial[\chi]_1(t,\mathrm{S})}{\partial S}\mathrm{d}S\approx[\chi]_1(t,\mathrm{S}_m)-$$
$$-[\chi]_1(t,\mathrm{S}_{m-1})=[\chi]_{1m}(t)-[\chi]_{1m-1}(t). \quad (14)$$

Substituting expressions (13) and (14) into equation (12), we obtain an equation for changing the value of the backlog of the m operation

$$\frac{dV_m(t)}{dt}=[\chi]_{1m-1}(t)-[\chi]_{1m}(t), \quad (15)$$

which is the level equation of the system dynamics model (1). Integration of the one-moment PDE model equation (6) within the m operation $S\in[S_{m-1}, S_m]$ allows the transition from PDE model equations to system dynamics equations. The rate equation can be obtained from the two-moment approximation [26]. This method makes it possible to use not a phenomenological approach, but a statistical approach when constructing the rate equations. At the same time, when constructing equations, exact model ideas about the statistical mechanism of interaction of parts with each other and with equipment as a result of technological processing are taken into account. This allows us to conclude that the system dynamics equations are a special case of the PDE model equations. The PDE model equations are the foundation for constructing level equations and rate equations in the system dynamics model.

Let us carry out the passage to the limit, demonstrating the relationship of the models, using the finite difference method. Let us represent equation (6) in the form of a finite difference equation

$$\frac{[\chi]_0(t_{i+1},S_m)-[\chi]_0(t_i,S_m)}{\Delta t}+$$
$$+\frac{[\chi]_{1\psi}(t,S_m)-[\chi]_{1\psi}(t,S_{m-1})}{\Delta S_m}+$$
$$+0(\Delta S)=0,$$

where $0(\Delta S)$ is the error of replacing equation (6) with a finite difference equation, $\Delta S=\max\{\Delta S_m\}$. Then the expression for the density of backlogs within the m technological operation takes the form

$$[\chi]_0(t_{i+1},S_m)=[\chi]_0(t_i,S_m)-$$
$$-\left([\chi]_{1\psi m+1}(t_i)-[\chi]_{1\psi m}(t_i)\right)\frac{\Delta t}{\Delta S_m}-$$
$$-0(\Delta S)\frac{\Delta t}{\Delta S_m}. \qquad (16)$$

Taking into account the limiting expression (11), we write

$$V_m(t_{i+1})=V_m(t_i)+\begin{pmatrix}[\chi]_{1\psi m}(t_i)-\\-[\chi]_{1\psi m+1}(t_i)\end{pmatrix}\Delta t-$$
$$-0(\Delta S)\Delta t. \qquad (17)$$

Using the recurrence relation (17) for a constant operation rate of equipment, we obtain the level equation of the system dynamics [8] in the form

$$V_m(t_{i+1})=V_m(0)+\begin{pmatrix}[\chi]_{1\psi m}-\\-[\chi]_{1\psi m+1}\end{pmatrix}t_i-$$
$$-0(\Delta S)t_i,$$
$$m=1\ldots M, \qquad (18)$$

determining the state of interoperational backlogs $V_m(t)$ for the m operation at time t_{i+1}.

6. Solving the problem of serial line functioning

Consider the problem of the functioning of a production line consisting of M technological operations with the initial distribution of parts over interoperational backlogs

$$V_m(0)=V_S(S_m),$$
$$V_S(S)=A\left(1+0.5\sin\left(2\pi S/S_d\right)\right),$$
$$S_m=S_d\cdot m/M, A=10^4, \qquad (19)$$

and a constant rate of processing parts at the m operation

$$[\chi]_{1\psi m}=[\chi]_{1\psi S}(S_m),$$
$$[\chi]_{1\psi S}(S)=MB\left(1+0.5\cos\left(2\pi S/S_d\right)\right),$$
$$B=2. \qquad (20)$$

The division of the technological route into an additional number of technological operations leads to a decrease in the execution time of a separate operation $\Delta\tau_{\psi m}$, and, accordingly, to an inversely proportional increase in the value of the operation rate $[\chi]_{1\psi m}=\Delta\tau_{\psi m}^{-1}$. When dividing each operation of a technological process consisting of M technological operations is into two operations with equal execution time

$$\Delta\tau_{\psi m\ 1}+\Delta\tau_{\psi m\ 2}=\Delta\tau_{\psi m},$$

$$\Delta\tau_{\psi m\ 1}=\Delta\tau_{\psi m\ 2}=0.5\Delta\tau_{\psi m}=$$
$$=0.5[\chi]_{\psi m}^{-1},$$

a new technological route consisting of $2M$ operations has the rate values for a separate technological operation $[\chi]_{\psi m}=2\Delta\tau_{\psi m}^{-1}$. This determined the presence of the factor M in conditions (19), (20). To construct a solution to the problem (5), (16), we introduce dimensionless parameters [27, 28]:

$$\xi=S/S_0,\ \xi_m=S_m/S_0,$$
$$\Delta\xi_m=\left(S_m-S_{m-1}\right)/S_0,$$
$$\tau=t/T_0,$$
$$n_m(\tau)=V_m(t)/V_0,$$
$$n_S(S)=V_S(S)/V_0,\ \theta_{1m}=\theta_{1S}(\xi_m),$$
$$\theta_{1S}(\xi)=[\chi]_{1\psi S}(S)/[\chi]_{1\psi 0},$$
$$\theta_0(\tau,\xi)=[\chi]_0(t,\mathrm{S})\frac{S_0}{[\chi]_{1\psi 0}T_0},$$
$$\theta_{0S}(\xi)=[\chi]_0(0,S)\frac{S_0}{[\chi]_{1\psi 0}T_0}. \qquad (21)$$

As the characteristic parameters of the production line model, we use the length of the technological route in the state space S_0, the average number of parts in the bufferV_0, and the average processing rate of parts for M operations $[\chi]_{1\psi 0}$. The scale of the parameters was chosen for the convenience of performing calculations [27, 29]. Due to the arbitrariness of the choice, we define T0 $T_0=V_0/[\chi]_{1\psi 0}$, $S_0=S_d$, which allows us to interpret the value of T_0 as the time during which the average interoperational backlog V_0 will be processed with the average rate $[\chi]_{1\psi 0}$. Solution (4), corresponding to the system dynamics model with initial conditions (19), (20) taking into account dimensionless parameters (21), can be written as follows:

$$n_m(\tau)=1+0.5\sin\left(2\pi\xi_m\right)+$$
$$+0.5M\left(\cos\left(2\pi\xi_{m-1}\right)-\cos\left(2\pi\xi_m\right)\right)\tau, \qquad (22)$$

$$n_S(\xi)=1+0.5\sin\left(2\pi\xi\right),$$
$$\theta_{1S}(\xi)=M\left(1+0.5\cos\left(2\pi\xi\right)\right), \qquad (23)$$

where $V_0=A$, $[\chi]_{1\psi 0}=B$, $n_m(0)=n_S(\xi_m)$, $\theta_{1m}=\theta_{1S}(\xi_m)$. Using the formula for the cosine difference and taking into account that for $M>>1(\xi_{m-1}+\xi_m)\approx 2\xi_m$, $(\xi_{m-1}-\xi_m)\approx\xi_m=M^{-1}$, it follows that

$$n_m(\tau)=n_m(0)+\sin(2\pi\xi_m)\pi\tau=1+$$
$$+\sin(2\pi\xi_m)(0.5+\pi\tau). \qquad (24)$$

We represent the solution obtained using the PDE model equations in dimensionless form

$$\theta_0(\tau,\xi)=\theta_0(0,\xi)-\frac{\partial\theta_{1S}(\xi)}{\partial\xi}\tau=$$
$$=\theta_0(0,\xi)+\pi M\sin(2\pi\xi)\tau. \qquad (25)$$

Taking into account the dimensionless notation (21), we write

$$n_m(\tau)=\theta_0(\tau,\xi_m)/M,$$
$$\theta_0(0,\xi_m)=Mn_m(0)=Mn_S(\xi_m), \qquad (26)$$

whence

$$\theta_0(\tau,\xi)=M(1+\sin(2\pi\xi)(0.5+\pi\tau)),$$
$$\theta_0(0,\xi)=\mathrm{M}(1+0.5\sin(2\pi\xi)). \qquad (27)$$

The transition from the dimensionless density of the distribution of interoperative backlogs $\theta_0(\tau,\xi)$ to the distribution of backlogs by buffers $n_m(\tau)$ is carried out in accordance with (27)

$$n_m(\tau)=\theta_0(\tau,\xi_m)/\mathrm{M}=1+$$
$$+\sin(2\pi\xi_m)(0.5+\pi\tau). \qquad (28)$$

Solution (28) coincides with solution (24). Within the specified time, the modeling of changes in the state of interoperational backlogs for a production line with a different number of operations M={500,100,20,10,5} is shown in Fig. 2. Fig. 3 shows the relative error of replacing the equations of the system dynamics model with the equation of the PDE model (τ=0.1).

$$\Delta n_m=0.5M\begin{pmatrix}\cos(2\pi\xi_{m-1})-\\-\cos(2\pi\xi_m)\end{pmatrix}\tau-$$
$$-\sin(2\pi\xi_m)\pi\tau. \qquad (29)$$

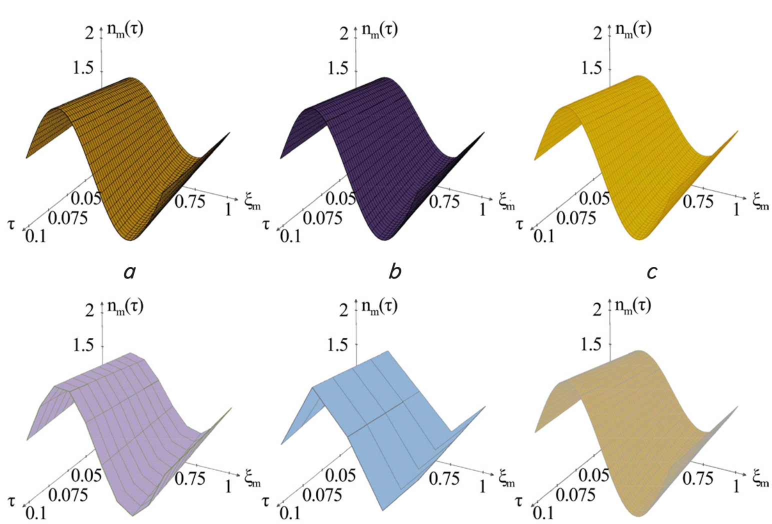

Fig. 2. Modeling the state of interoperational backlogs: *a* – M=500; *b* – M=100; *c* – M=20; *d* – M=10; *e* – M=5; *f* – PDE

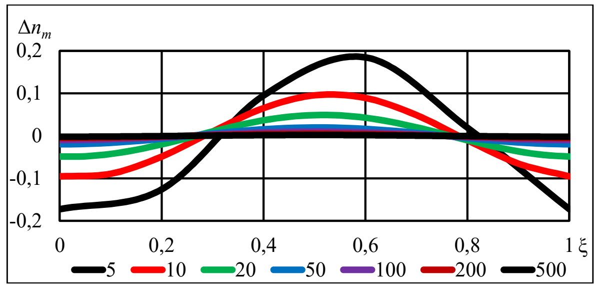

Fig. 3. Relative deviation of interoperational backlogs Δn_m, *M*={500, 100, 20, 10, 5}

The difference between the solutions using the system dynamics model equations and PDE model equations is observed for small values of *M*={20,10,5}. With a large number of operations, the use of the system dynamics equations and PDE model equations leads to the same solution, which is a consequence of using the limiting relation $\Delta\xi_m=M^{-1}<<1$.

7. Discussion of the results of research on methods for describing production lines

The paper substantiates and presents a criterion for the limit transition from the system dynamics equations to the PDE model equations, which allows us to consider the system dynamics equations as the limit case of the PDE model equations when describing production multi-operation lines.

A strong limitation of the system dynamics model as a general apparatus for the macroscopic description of a production system is that the system dynamics equations contain phenomenological coefficients that determine the relationship between the amount of interoperational backlogs and the flows of parts entering and leaving the technological operation. The presence of coefficients is due to the use of a phenomenological approach to construct the system dynamics equations. The PDE equations are based on another principle based on the fact that the production line is considered as a system consisting of a large number of parts that are in different stages of production (work in progress). Despite the fact that the processing of parts along the technological route obeys strictly specified laws of the technological process of product manufacturing [1], the presence of such a number of parts leads to the emergence of strict dependencies between the macroparameters of the production line – the backlog amount and the processing rate of parts in a technological operation. These dependencies take into account the structure of the technological process and can be determined for stationary and transient modes of production. Thus, the substantiation of the limit passage allows using the PDE model as a tool for constructing phenomenological coefficients for transient operation modes of production lines, both in the absence and in the presence of feedbacks. The proposed tool provides effective methods for solving the problem of accounting for the structure of reserves and flows [13–15] when constructing system dynamics equations.

The results of solving the functioning problem of a serial line [6] with the number of technological operations M={500, 100, 20, 10, 5} are shown in Fig. 2. The characteristic time for analyzing the process dynamic characteristics (28) is determined by the inequality $\tau_d \sim (2\pi)^{-1}$. Criterion M^{-1} is the basis for choosing a model for describing a production line with a sequential order of technological operations (Fig. 1), determines the deviation between the results of calculating the interoperational backlogs of a production line (Fig. 2), performed using the system dynamics equations (22) and PDE model equations (28). The deviation shown in Fig. 3 in accordance with expression (29) has the estimate $\Delta n_m \sim M^{-1}$. For $M^{-1}>0.01$, it is recommended to use the system dynamics model equations to describe the production line. Otherwise, it is preferable to use the PDE model equations. The choice of the criterion value is determined by various factors that determine the features of production line modeling, among which the time complexity of the model implementation algorithm is of no small importance.

The transition criterion M^{-1} defines the constraint for replacing the system dynamics equations with the PDE model equations. It should be noted that this limitation applies to production lines with a non-linear dependence of the technological equipment productivity on the number of technological operations at a given moment in time. For synchronized or quasi-synchronized production lines, due to the form of equations (6), (9), this limitation is absent. The transition is possible for an arbitrary number of technological operations. The limitation also does not apply to the case of production lines with a linear dependence of the technological equipment productivity on the number of technological operations. The remark is quite important due to the fact that the standard operation mode of the production line is a synchronized or quasi-synchronized mode [10].

The prospect of further research is the development of methods for constructing similarity criteria of production systems, taking into account both the number of technological operations and the operating mode of the production line, in order to justify the choice of a production line model.

8. Conclusions

1. The features and conditions of applying the system dynamics model and PDE model equations for describing production lines are considered. It is shown that with an increase in the number of technological operations, the duration of calculating the production line parameters using the system dynamics model equations can exceed the allowable time required to ensure control actions. For the considered serial-line model with a time-independent processing intensity of a product for a technological operation and the absence of feedbacks, the processor time costs linearly depend on the number of system dynamics equations. The presence of feedbacks caused by control actions leads to a nonlinear dependence of the computing resources costs.

2. The limiting transition from PDE model equations to system dynamics equations for $M>>1$ is presented, showing the relationship of the models, suggests that the system dynamics equations are the limiting case of the PDE model equations. The conditions for choosing a model for describing a production line with a sequential order of technological operations are determined by the value of the criterion M^{-1}, which characterizes the number of production operations in the technological route. A comparative analysis of the solution of the problem of the production line operation is carried out, which shows that with a large number of technological operations, the system dynamics equations can be replaced by the PDE model equations.

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